NAVIGATION BY BEARING INFORMATION

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ABSTRACT

A computer simulation model was developed and a series of experimental runs was conducted to investigate the technique of navigational fixing through the use of bearing information alone. It was determined that system bearing accuracies on the order of ± 0.50 degree will give fixing accuracies of 15 to 30 yards at ranges of 2000 to 3500 yards, less than 150 yards out to five miles (10,000 yards) and on the order of 200 to 250 yards out to ten miles, the maximum range considered. This is considered to be sufficiently accurate to be of value in the solution of the local area navigation problem.

Modern aircraft systems, capable of generating an inertial quality heading reference and containing a digital data processing system, are capable of overall system bearing accuracies of approximately ± 2.0 degrees. The technique of bearing averaging; i.e., using as a bearing the mean value of a series of rapidly measured and computed bearings, upgrades the accuracy of a ± 2.0 degrees system to approximately that of the ± 0.50 degree system. It is therefore concluded that bearings-only fixing as an aid to local area navigation is a feasible technique for use with the latest digital aircraft systems. Recommendations are made for eventual implementation of the bearings-only technique in an operational system.

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CHAPTER 1

INTRODUCTION

1.1 Navigation at Sea

Since man first ventured out to sea, navigation has been of continual and critical importance to him, both in successfully carrying out his mission and in returning safely to port. The basic navigation problem for an operational (military) aircraft or ship at sea in the world of today resolves itself into two separate and separable problems; that of "absolute" navigation and that of "relative" navigation.

Absolute navigation is here defined as navigation with reference to a large scale generally accepted coordinate system, such as Latitude and Longitude. When navigating in such a system a vehicle is able readily to communicate its position to others directly in terms of the reference coordinate system, to determine its own position relative to known fixed locations (Boston, Mass., Johnson Island, Greenwich, England) and to determine its position relative to others utilizing the same system.

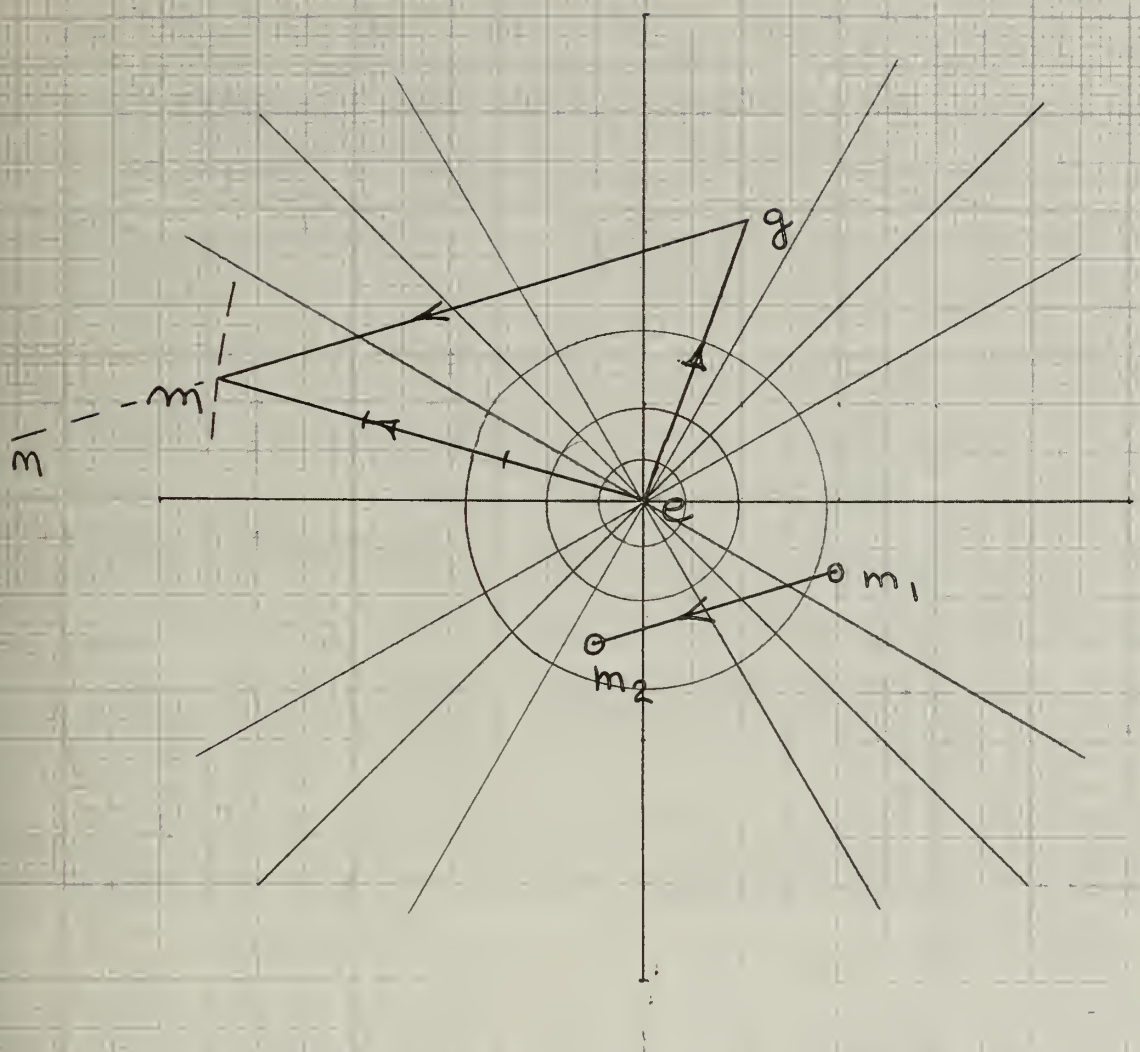
Relative navigation is defined as navigation relative to other ships, aircraft, floating sensors, and targets in

a local area covering at most a few hundred square miles. A local coordinate frame, frequently a simple square grid, is used and its relation to a Latitude, Longitude reference frame or to any other coordinate system is neither important nor is it necessarily fixed. It is to this second or "relative" navigational problem and its improved solution that this thesis addresses itself.

1.2 Local Area Navigation

Local area navigation has always been of prime importance to the sailor. In the earliest days, man navigated timidly from point to point, using visual reference points and "feeling" his way along in a known, familiar frame of reference. Later, with the advent of modern navies, ships sailed the oceans of the world and were maneuvered in close company by means of relative navigation and with the aid of such devices as a maneuvering board; i.e., a simple plotting board used to compute relative motion. A simple maneuvering board problem is pictured in Figure 1, and its use is described for the would-be-navigator in Chapter XIX of Reference 1.

Relative or local navigation, by whatever means accomplished, always reduces to the following basic problem; a localized or tactical situation must be oriented relative to a designated reference direction, generally North, and all participants be they ship, aircraft, enemy target, floating buoy or underwater sensor must be "fixed" or



Problem:
Guide: on course 020°, speed 12 kts (line e-g)
Your initial position = stbd beam of guide, distance 4 miles (point m_1)
Problem: take up position 3 miles astern of guide (point m_2) using 18 knots speed.
Solution: draw line g-n parallel to m_1-m_2. Swing radius 18 kts from e till it intersects g-n.
Answer: line e-m, your correct course 286°.

FIGURE 1

SIMPLE MANEUVERING BOARD PROBLEM

"accountable" within that frame. It is not mandatory that this local coordinate frame remain fixed relative to some outside frame, indeed this frequently is not the case; it is only required that all participants be accountable in this local frame for the duration of the problem, situation or attack.

As mentioned previously, relative navigation or station keeping among ships is an art which has been developed over the years. The advent of radar in World War II supplemented the maneuvering board and other means of local navigation, and helped to overcome that last bugaboo of the ship's navigator, fog and reduced visibility. For aircraft, however, the problem remained a severe one.

An aircraft operating independently has a much more difficult navigation problem than does a ship. The effect of wind on his flight path may be an order of magnitude greater than the corresponding effects of tide and current on a ship's movement through the water. His aeronautical bubble octant is less accurate than is a ship's sextant, it is used under adverse circumstances of motion and vibration and it is subject to acceleration effects not present, or negligible, aboard ship. Use of an aircraft bubble octant is described in Chapter XX of Reference 1. In the local or tactical area, the aircraft frequently has no reference other than one which he himself has thrown into the water. Orienting and fixing the aircraft in the local area initially is difficult at best,

and maintaining this local area "stabilized" for any period of time is frequently impossible.

One of the most widely used sensors of a modern anti-submarine warfare (ASW) aircraft is the sonobuoy. The sonobuoy is dispensed from the aircraft and is considered expendable. It consists of a cylinder about three feet long (high) and some three inches in diameter, floating with just a few inches out of the water. Suspended below the buoy proper is a hydrophone, and noises received from the surrounding water are transmitted by radio to the parent aircraft. As this buoy, or a number of these buoys in the same local area, may be the only link between the aircraft and a submarine target, accurate local area navigation relative to such a floating buoy is both a pertinent and an important requirement in the Navy today.

Navigation of an aircraft relative to a floating object, such as a buoy, requires stabilization of the aircraft navigational plot relative to that buoy. If there were no errors in the aircraft navigational and computing equipment, and if the buoy were stationary, then all that would be required to stabilize (i.e., solve the problem) would be an accurate determination of the wind vector. However since the buoy does move slowly with wind and current and since the predominantly analog aircraft equipments do contain built-in errors, obtaining a solution is not that simple and straightforward. A reasonable operating procedure is as follows. With the best estimate of wind

set into the aircraft navigation system the aircraft is flown over the desired buoy, and visually marked "on top." Marking "on top" consists of the pilot maneuvering the aircraft so as to pass as closely as possible directly over the buoy, and then estimating visually the exact instant of passing overhead. A position obtained in this manner may easily be in error by from 30 to 50 yards. A few minutes later a second "on top" is marked, and the plotted or remembered position of the buoy is noted to have moved relative to the actual position. By noting the elapsed time and the magnitude and direction of apparent buoy motion a correction factor can be computed and applied to the assumed wind vector. The new "system wind," which includes buoy motion, aircraft system equipment errors and the wind itself, should now stabilize the local area plot. The disadvantages of such a system are obvious; it is slow and time-consuming, and it requires frequent visual over-flight of the buoy. Additionally, when using an aircraft system containing such analog equipments, the errors present are variable and unpredictable. System errors build up quickly, necessitating frequent buoy revisiting for updating and re-stabilizing. Unfortunately, the very nature of such an analog system, with its variable and unpredictable errors, makes navigation by bearings only (without over-flight and visual sighting) not a feasible technique; "on-top" remains a requirement in order to maintain a stabilized local reference.

1.3 New Developments

By addition of a doppler-inertial navigation system, the overall aircraft capability may be improved significantly. Inertial quality heading information from the inertial platform plus accurate track and ground speed values from the doppler radar are now available continuously to all portions of the aircraft system. Given an opportunity to visually mark "on-top" and thus determine "system wind," such a aircraft system can be stabilized quite accurately. Retention of analog portions of the system still causes the solution to deteriorate, however, and a new solution is required periodically. Replacement of the analog equipments with a digital data processing system would not only greatly reduce system error, but would also permit accurate second-by-second determination of aircraft position combined with a very high speed digital computation capability.

Such a total system capability exists in the A-NEW avionics system, which is described in some detail in Reference 2. The A-NEW system is currently being installed in the U.S. Navy's P3C Orion land based ASW aircraft. The P3C aircraft contains a doppler-inertial navigation system capable of providing highly accurate positional and heading outputs. The data processing system is built around a high-speed, general-purpose, stored-program digital computer. Tactical sensor information is first acted upon by tactical crew members, the information is then

processed and correlated by the digital system, and the output is presented quickly and accurately on digitally generated cathode ray tube displays for action by the tactical crew. One of the most significant advantages of the A-NEW system established in developmental flight testing, and as reported in Reference 2 which describes the entire system, is greatly improved navigation compared to other operational systems. With a system such as A-NEW, which is in the P3C and will be the avionics system for the Navy's next generation carrier-based ASW aircraft, designated the VS(X), the bearings-only technique appears to be a feasible and potentially valuable addition to the local navigation system.

1.4 Previous Work

Because of the large and variable system errors and the comparatively slow data rates possible with analog systems, as discussed in the previous section, no serious consideration has been given to use of the bearings-only technique in such operational aircraft systems. A conventional loop antenna is used for radio direction finding, and this same loop is used in homing on buoys until within visual range of "on top." Some consideration has been given to the possibility of bearings-only positioning with an A-NEW type system. Most study effort to date has concerned itself with various schemes of fixed antenna arrays, methods of obtaining usable antenna

patterns, electronic switching techniques and similar technical questions. Specific results are not available in this thesis because of security classification. The P3C A-NEW system, currently going into production and described in Reference 2, does not incorporate a bearings-only local area positioning capability.

1.5 Method of Attack

Quite apart from specific "hardware" considerations such as the number of antennae required and their possible locations a valid area of inquiry exists as to what would constitute a "useful" addition to existing aircraft capabilities. If system bearing accuracies of ± 5.0 degrees are obtainable, is this capability of any use operationally? If a system bearing accuracy of ± 0.1 degree is required in order to be "useful" can this be obtained within today's state of the art? Modern high-speed digital computer techniques make simulation of such systems possible. A large number of simulated flights can be "flown" on the computer in a relatively short time. System bearing errors can be bounded precisely and varied over a range of values from unacceptably large to extremely small. Corresponding local area bearings-only fixing errors can be computed. Conclusions may be drawn concerning the operational usefulness of systems having the characteristics of those simulated. Appropriate recommendations can then be made, intelligently, as to the feasibility of acquiring

a given capability or the desirability of attempting to fabricate systems having desired characteristics.

CHAPTER 2

PROBLEM DEFINITION AND SIMULATION

2.1 The Operational Situation Simulated

The operational situation modelled is that of a single aircraft operating independently in the open ocean on an ASW-type mission. A floating buoy is presumed to be the reference for local area navigation. A ship-to-buoy situation could have been presumed just as well, but the aircraft-buoy situation is both more stringent and of greater personal interest to the author. The aircraft is presumed to have the capabilities ascribed to the P3C A-NEW system in Reference 2. All tactics and capabilities suggested in this section or elsewhere in this thesis are considered to be representative of those of a class of aircraft and aircraft systems, and are not to be construed as representing any one specific existing aircraft or aircraft system.

Aircraft speed is maintained at 180 knots, a reasonable assumption for most aircraft, and all turns are made at 3 degrees per second (which is considered to be standard rate) or less. Local area is defined to extend to a range of ten nautical miles (20,000 yards) from the

buoy, which is defined to be at datum, and bearings-only fixing is assumed of interest from that maximum range to a minimum of one mile (2000 yards) from the buoy. At one mile the aircraft could close the datum in a maximum of 20 seconds, and no appreciable system error will accumulate in that time.

The digital (A-NEW) system is presumed to be programmed to sample bearings from the aircraft to the buoy at intervals of 20 seconds. Each such bearing, at the aircraft computed position, is cross-plotted with the last previous sample which has been held in memory. Were there no errors these two bearing lines would intersect at the actual location of the buoy. The system is presumed programmed to hold the last three bearings in memory at any given time. Logic is provided to compare each pair of selected bearings before cross-plotting. If their angular difference is less than 15 degrees then by definition they do not constitute an acceptable pair, and another stored bearing is selected to compare with the current one. This procedure is repeated until a pair of bearings is found differing by 15 degrees or more, and these bearings are then cross-plotted.

Figure 2a shows schematically how inputs from the various elements of the aircraft navigation system are combined in the solution of the local area navigation problem. The inertial system provides a heading reference, not necessarily true North, but constant over the

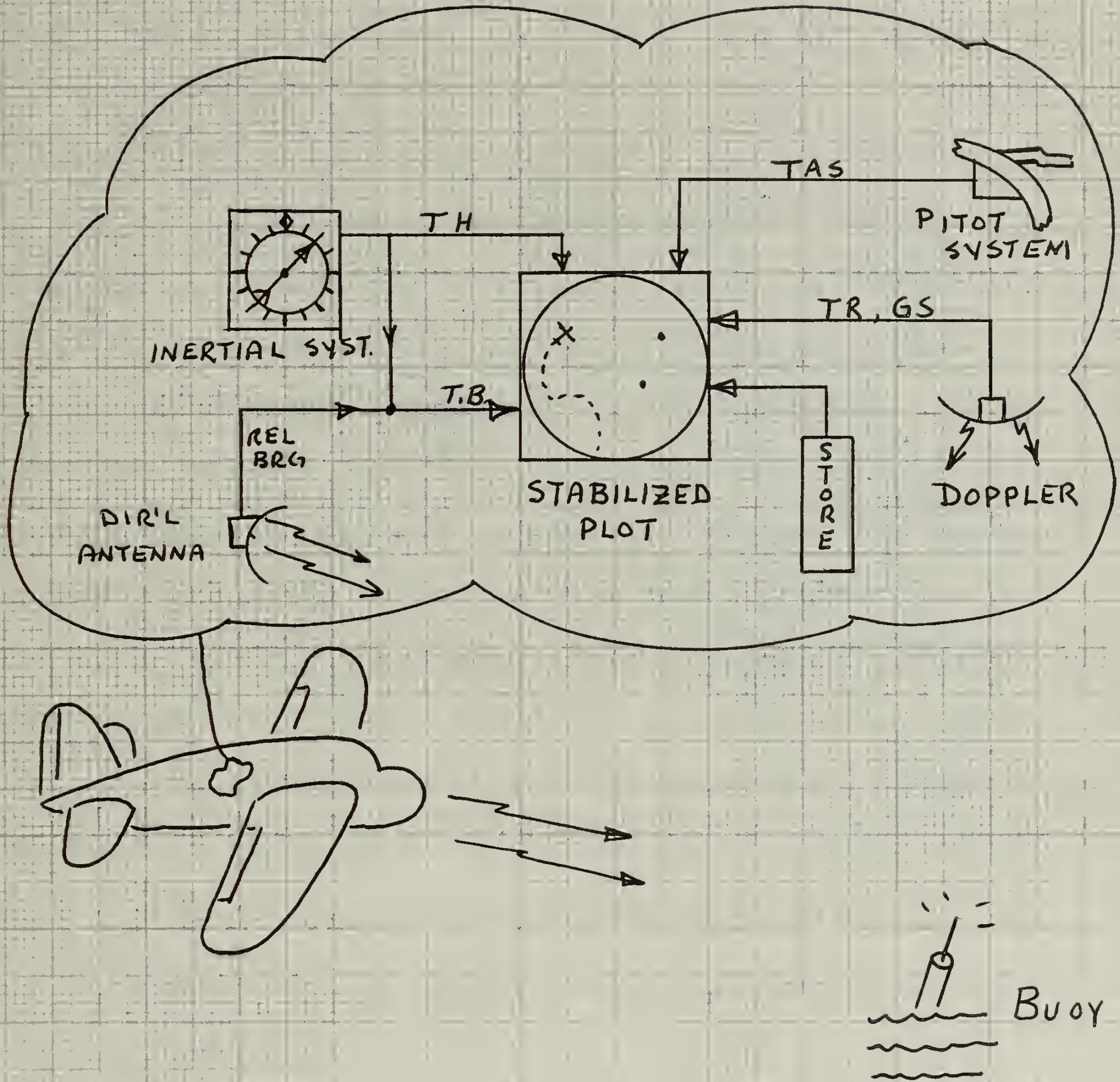


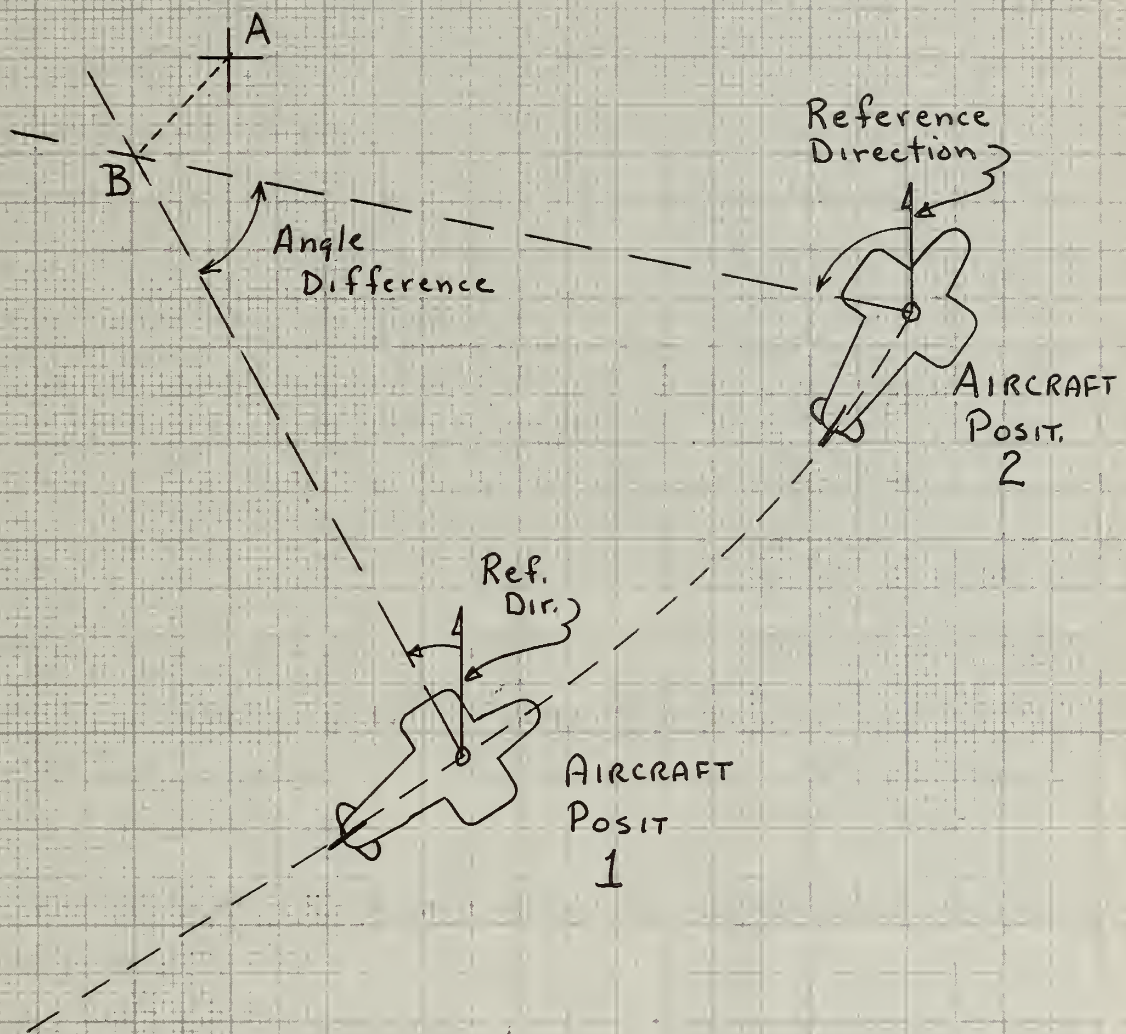
FIGURE 2a

TYPICAL AIRCRAFT NAVIGATION SYSTEM

short time of the local area problem. The doppler radar provides aircraft track and ground speed; this combined with true air speed from the pitot static system and inertial heading constitute the source of the initial wind vector. These inputs together allow ground stabilization of the tactical display so that geographic features (and the local area frame of reference) remain fixed while the aircraft and other moving objects proceed realistically across the display. This display stabilization also allows the aircraft system to "remember" the position of an object, and this in turn is the key to comparing new positional information (from bearings cross-plotted, for example) against "old position" as a measure of system error and/or changes in the movement of the referenced buoys.

It should be noted that the term "ground stabilization" is really a misnomer in this case, as it implies a local area coordinate system fixed in earth coordinates. The local area navigation system is in fact referenced to a slowly moving floating buoy, and therefore more correctly "water motion stabilized." The doppler radar, of course, also measures "ground speed" and "ground track" relative to the water surface over which the aircraft is flying, not relative to the earth.

Figure 2b illustrated diagramatically how a bearing pair such as has been described above might be used as part of such an aircraft navigation system to obtain a fix.



Point A: Remembered Buoy Position
Point B: Bearings-Only Fix
Distance A-B: System Drift

FIGURE 2b

BEARINGS-ONLY FIXING TECHNIQUE

2.2 The Experimental Situation

The experimental runs were conducted using the M.I.T. Compatible Time Sharing System (CTSS). Time-sharing is an ambiguous term, frequently used to describe the concurrent operation of several parts of a single computer. The M.I.T. time-sharing system, CTSS, is one which has as its goal concurrent, effective use of a single large digital computer by a number of clients. CTSS is a general purpose programming system which allows this new "shared" technique of computer operation and yet permits users to continue to utilize existing programming systems. CTSS is used from a console which is essentially an electric typewriter. The console user controls the computer by issuing standard commands, one at a time, which provide for most of the usual routine programming operations, or which may call in an arbitrary programming subsystem (such as FORTRAN) with its own control language. Each console user is serviced in short "bursts" of computer time, on a sequential basis, and therefore the individual user pays no charge for time spent typing (or thinking), nor is valuable computer time wasted in waiting upon the individual user. The appearance of this system to the individual user is that of having the computer responding quickly to his commands as if he alone were using it, even though 25 or 30 other users might be operating concurrently. The CTSS is described in great detail in Reference 3. Reference 4 contains the minimum

information needed to allow a new user to operate the system.

In using the CTSS for this thesis, a pseudo-random flight path was simulated, as shown in Figure 3. This flight path originated at ten nautical miles (20,000 yards) from the reference buoy, and proceeded by various courses to a minimum range of one mile (2000 yards) before beginning to open the range again. During this process, the aircraft covered a wide variation in range, bearing, range-rate and bearing-rate; the constraints previously spelled out for the operational situation were observed.

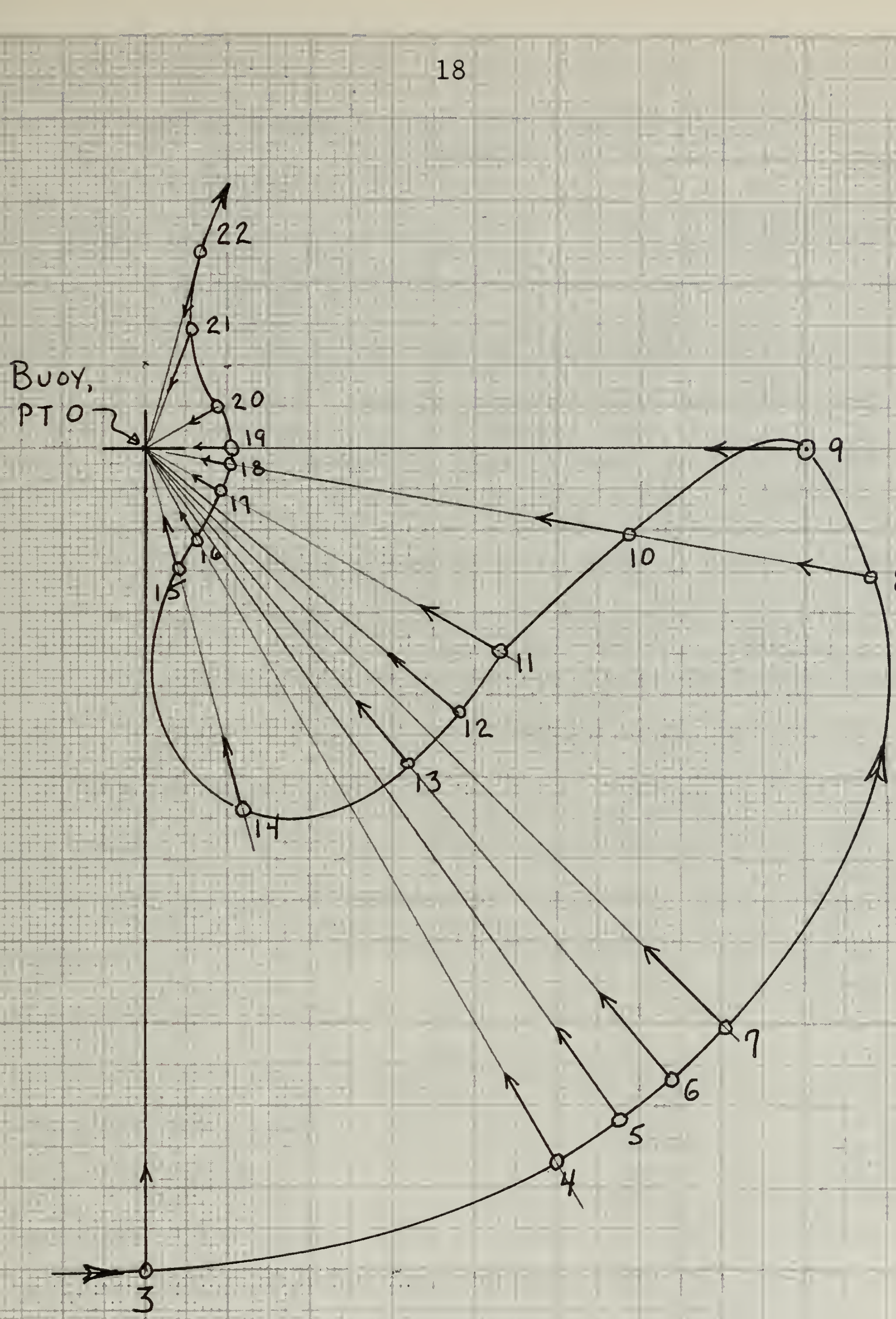
A series of finite points was specified along this flight path, differing in bearing from the reference buoy by angles of from 5 to 60 degrees. This basic flight path and the selected points along it were used for all of the experimental runs conducted.

The following conventions were followed in numbering points along the flight path and those points formed by the intersection of two bearing lines:

(a) The reference buoy was located at the origin of the coordinate system, and was designated Point 0.

(b) Reference points along the flight path were numbered sequentially from Point 3 to Point 22.

(c) Cross-bearing intersections were numbered by combining the numbers of the two points from which the bearing lines originated; i.e., the point defined by the intersection of bearings



NOTE: X-Y coordinates of all reference points, bearings to Point O, and distance from Point O are tabulated in Appendix B.

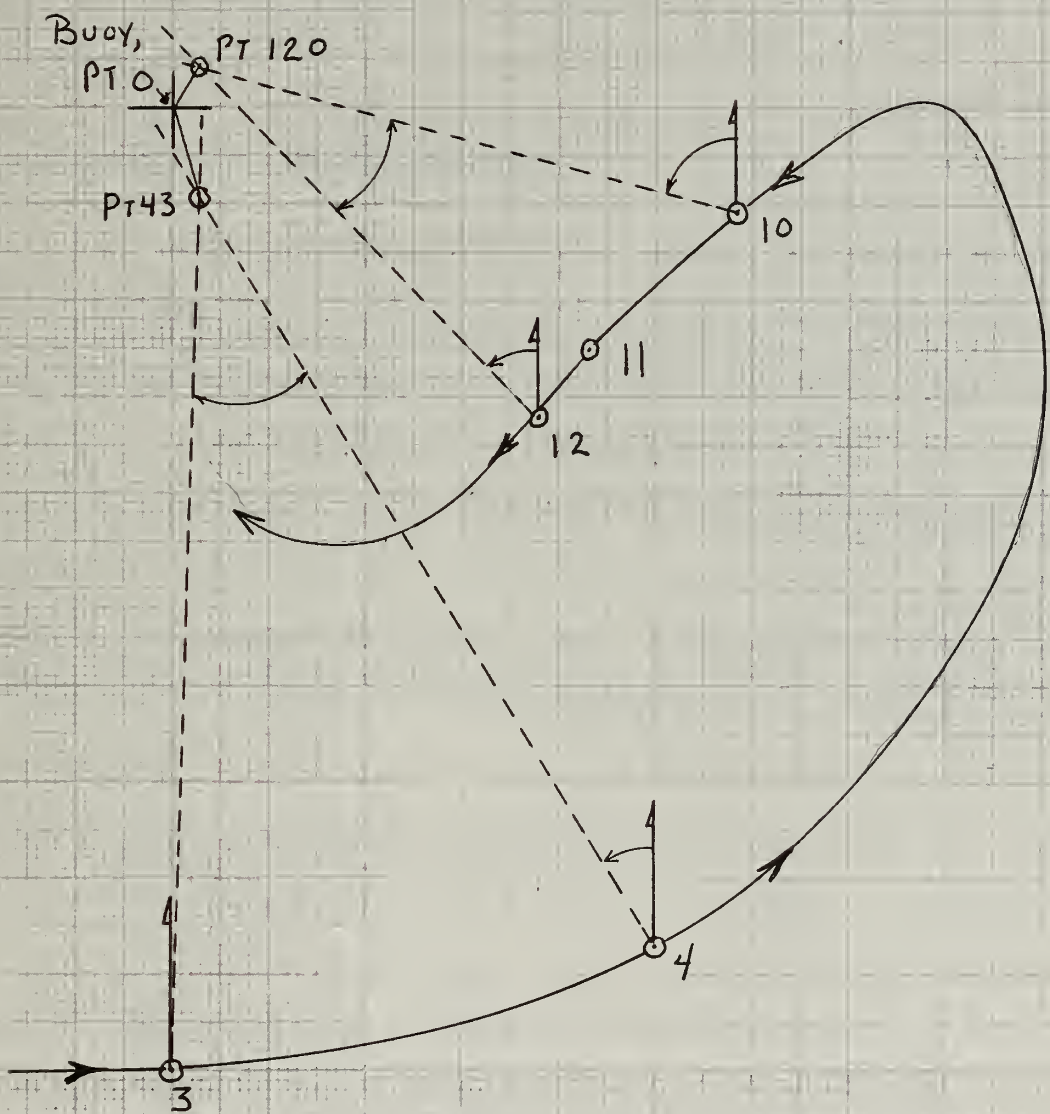
FIGURE 3

BASIC SIMULATED FLIGHT PATH

from Point 7 and Point 4 was designated Point 74.

(d) Because of a convention in the programming language used, point numbers were restricted to no more than three digits; i.e., points 0 to 999. Points such as 1715 and 2017 were therefore arbitrarily reduced to 175 and 207, respectively, by omitting the first digit of the second point in each case. No ambiguous point numbers resulted from this convention. Figure 4 illustrates the plotting and numbering of two typical intersections.

The computer language used in conducting this research was COGO-90. COGO (Coordinate Geometry) is a problem-oriented language developed by the Civil Engineering Department at M.I.T. It was developed primarily for use of civil engineers and as such is based on a rectilinear coordinate system. The commands available include those for determining distances, angles, bearings, included areas and other geometric values. The language is simple and straightforward; it was designed with the goal in mind of making the computer available for the use of a civil engineer, without the necessity for special training. The basic geometric format of COGO and the ability to perform functions such as storing points, computing intersections of bearing lines and measuring distances make it particularly useful for this experimental application. A basic description of



NOTES:

1. Reference direction is approx. North.
2. Distance 0-43 and 0-120 are measures of fixing error.

FIGURE 4

TYPICAL BEARINGS-ONLY FIXES

COGO-90 and its use with CTSS is contained in Reference 5. Appendix A contains a simple COGO-90 run as conducted using the CTSS system. The points chosen and values used are taken from a typical experimental run made during the course of the present research. Comments are included to assist the reader.

2.3 Scope of the Experiment

It was desired to conduct a series of simulated flights, taking bearings to Point 0 from each of the previously chosen reference points along the flight path. For each flight, or set of flights, bearing errors up to a predetermined maximum value were introduced as system error or "noise" superimposed on each bearing measurement. Specific error for a given data point was determined by selecting a number from a table of 2500 random numbers, Table 26.11 of Reference 6, and then applying that error value to the basic known bearing from the data point to Point 0. This procedure applied in turn to each of points 3 through 22 make up a table of bearings for a given experimental flight (run), each bearing containing a random amount of error up to the maximum value for that particular run. This technique was used to prepare input data for ten experimental runs each at maximum system errors of ± 5 degrees, ± 2 degrees, ± 0.5 degree and ± 0.25 degree.

For each experimental run, the flight was simulated to proceed successively from Point 3 to Point 22. At each

data point the criteria described previously (at least 20 seconds time interval and 15 degrees or more angular difference) were applied to determine whether a "valid" cross-bearing could be plotted using the last previously measured bearing. If a valid cross-bearing did not exist, the next previous bearing was selected and the criteria applied again, and so on until an allowable pair of bearings had been selected. For the assumed reference points, 3 through 22, and for the criteria established above, sixteen valid bearing-pairs were plotted on each experimental run, resulting in sixteen fix positions each different from Point 0 by a distance directly interpretable as fixing error. Appendix B contains the complete input data required for one set of experimental runs, namely ± 2.0 degrees. Appendix C is extracted from the computer printouts of the same set of runs, and contains the X and Y coordinates of all sixteen points plus the corresponding fixing errors or distances computed from Point 0.

One possible technique considered for improving or upgrading a given quality of input was that of averaging. If a series of bearing measurements were to be made at very short time intervals and the arithmetic mean computed, then that mean bearing could be considered to be the bearing of Point 0 from the aircraft's position at the mean time. The quality of such a "mean-bearing" would be improved over that of the individual bearings. In a digital system such bearings could easily be taken at 0.5 to 1.0

second time intervals, and a series of ten or more could be used to arrive at an average value. A limited number of experimental runs were made simulating this technique. Ten random numbers were taken from Table 26.11 of Reference 6, the arithmetic mean computed and this number used as the "error value" to be applied to the bearing from one experimental point for one run. The entire procedure was repeated for each experimental point required. This experimental technique of error averaging and then applying the arithmetic mean error to a single bearing produced mathematically the same result as would have been accomplished by averaging a group of ten bearings, each containing some previously applied random amount of error within the prescribed limits, and then plotting the mean bearing thus obtained. A series of these so-called "averaged" runs was made for single value bearing errors of ± 2.0 degrees and ± 0.5 degree. The results obtained from these averaged runs are tabulated relative to the results of the various runs made using single bearings. Comparisons are made and the relative improvement due to averaging is noted.

CHAPTER 3

INTRODUCTION

3.1 Basic Results

The first set of experimental runs was conducted using a maximum total system error of 10 degrees; i.e., plus or minus 5 degrees about the correct bearing, from the point in question to Point 0. This amount of system inaccuracy corresponds roughly to the overall bearing taking capability of typical current fleet aircraft navigational and radio direction finding systems. These aircraft contain predominantly analog devices, contain no digital data processing and may or may not provide inertial quality heading information. Under such circumstances it has been presumed previously (section 1.2) that the bearings-only technique would not prove itself a usable addition to the system. The results of ten experimental runs using maximum system bearing errors of ± 5.0 degrees indicates that the bearings-only technique is not usable for an aircraft having system errors of that magnitude. These results are shown in Table I. It is noted that mean fixing errors are of the order of hundreds of yards at long ranges, and even in at the closest points (one nautical mile from Point 0) they vary

TABLE I

FIXING ERROR FOR ±5.0 DEGREES BEARING ERRORS

Point	Angle	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8	Run 9	Run 10	Average
43	30	3576	1168	2594	*7311	2081	436	3309	4867	4079	1052	3047
53	35	856	2258	1837	*4109	1760	1642	4077	3651	1436	736	2236
63	40	2871	1107	*3956	1898	1473	2418	1495	916	1064	2206	1940
74	15	*6225	2487	5932	5344	2459	2205	1186	5223	2973	1404	3544
87	35	3315	2034	3527	*4040	1558	805	1981	3059	1291	1956	2357
97	45	853	1792	1556	*2529	605	1585	1196	420	959	671	1217
107	35	858	551	*2235	856	758	846	1187	944	666	1033	993
110	20	1496	1275	742	675	715	2743	*3711	1439	2189	1584	1657
120	30	1923	757	2129	932	650	1316	1783	684	1361	*4687	1622
131	20	1352	476	1749	1762	507	*6953	2936	2080	1591	1468	2087
143	25	*2743	560	2449	599	522	1229	1934	807	1883	1331	1406
153	25	1305	93	1404	2106	1259	1513	1391	*2859	798	777	1351
175	45	287	106	79	137	192	158	374	234	*434	247	225
207	60	84	168	138	172	203	164	65	107	115	*223	144
210	40	78	153	98	*243	115	*243	80	141	231	171	155
220	45	287	201	522	236	122	178	346	53	79	*628	265

1. Fixing error in yards.
2. *indicates largest value recorded for that point.
3. Angle indicates included angle in degrees.

between 150 and 250 yards. Individual errors as high as 7000 yards were recorded, and even after ten experimental "flights" individual error values had not begun to average out to anything resembling a consistent "mean value." Homing in on a buoy to a visual "on-top" using present techniques can give consistently better results than does this system when system bearing errors are of the order of ± 5.0 degrees.

The next ten experimental runs were made with maximum bearing errors limited to ± 2.0 degrees, or a total of 4.0 degrees maximum. This quality system is believed to represent an achievable capability within today's state-of-the-art; it should be realizable in a system such as that of the P3C A-NEW configuration described in Reference 2. Results at ± 2.0 degrees are shown in Table II. They are much improved over the results of the first set of runs. Mean fixing errors (after ten runs) for a given intersection were of the order of one-half to one-third as large as in the previous case at ± 5.0 degrees bearing error. For the reference points close in to the buoy, i.e., those at from 2000 to 3500 yards from Point 0, mean fixing errors were reduced to from 70 to 140 yards. At the longer ranges, out to ten miles (20,000 yds.), error on this set of runs was still measured in hundreds of yards. Fixing errors of that magnitude would be unacceptably large, causing much unnecessary system slewing and repositioning when the "reason" was really just an internal "system" problem.

TABLE II

FIXING ERROR FOR $\pm$2.0 DEGREES BEARING ERRORS

Point	Angle	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8	Run 9	Run 10	Average
43	30	965	956	814	*2243	705	2066	1347	779	663	1165	1170
53	35	836	539	*1567	704	580	558	733	1163	587	1273	854
63	40	513	498	618	*1259	849	1041	1196	284	202	657	712
74	15	2228	*5873	1346	458	2482	657	860	1312	328	2264	1781
87	35	294	1308	321	454	1319	743	608	*1750	663	1026	849
97	45	516	574	447	458	663	597	921	*1369	372	610	653
107	35	408	1125	650	465	1437	1388	753	*1635	421	528	881
110	20	*1998	830	608	314	939	1043	298	1426	1195	1081	973
120	30	449	837	327	*1447	133	500	205	1079	816	377	617
131	20	225	932	353	*2201	488	217	1503	344	56	445	676
143	25	887	982	*1116	959	719	592	357	552	357	527	705
153	25	397	516	405	*740	226	132	642	108	66	21	325
175	45	118	71	132	65	*194	65	116	162	93	35	105
207	60	91	*116	42	53	84	104	43	99	59	44	73.5
210	40	47	67	67	89	52	*141	76	135	58	38	77.0
220	45	62	162	205	230	127	*256	27	206	91	36	140.2

NOTES:

1. Fixing error in yards.
2. * indicates largest value recorded for that point.
3. Angle indicates included angle in degrees.

Such a capability is that just described might prove to be useful in at close ranges but most certainly does not produce consistently useful information at medium and long ranges, and its contribution to the overall system capabilities would probably not improve overall system performance sufficiently to justify its inclusion in a system.

A third set of data was obtained from ten more runs of sixteen intersections each. Once again allowable error was reduced, this time to $\pm$ 0.5 degree, a total maximum allowable of 1.0 degree. Table III displays the results of these ten runs. This tightening of maximum allowable bearing error again brought a corresponding reduction in fixing error of a factor of approximately four over the previous set of runs ($\pm$2.0 degrees). Mean fixing error values at close-in ranges were reduced to 15 to 30 yards, and errors of 150 yards or less prevailed out to 5 miles (10,000 yds.). Even at ten miles, the maximum range considered to be "local area," the average fixing errors were on the order of 200 to 250 yards. If it is assumed that the system is capable of discriminating against the occasional extreme sample, and such a filtering of information is easily accomplished with a digital system, this level of system bearing accuracy gives a fixing capability, as described above, which if incorporated into any current aircraft system would enhance significantly the local area navigation capability of that system.

TABLE III

FIXING ERROR FOR ± 0.50 DEGREE BEARING ERRORS

Point	Angle	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8	Run 9	Run 10	Average
43	30	275	198	414	153	354	87	*553	109	143	404	269.0
53	35	138	355	159	238	60	73	141	238	195	*455	205.2
63	40	210	140	*259	210	108	202	180	106	149	200	176.4
74	15	774	*1281	894	983	1016	560	1015	106	306	272	720.7
87	35	459	141	390	334	126	*537	397	106	176	106	277.2
97	45	*383	152	149	256	287	175	141	325	336	108	231.2
107	35	203	302	105	*365	320	247	166	105	189	216	221.8
110	20	240	99	287	240	200	44	275	240	*418	216	225.9
121	30	137	114	165	195	202	78	*282	137	245	39	159.4
131	20	202	71	100	*260	204	152	56	103	199	199	154.6
143	25	70	228	146	170	90	40	110	*233	195	146	142.8
153	25	181	151	86	*222	101	30	22	38	146	18	99.5
175	45	11	20	41	15	28	26	27	*44	41	22	27.5
207	60	25	*29	11	4	11	15	12	14	15	4	14.0
210	40	*58	16	34	5	7	16	0	16	21	13	18.6
220	45	19	29	33	34	10	25	12	49	*75	22	30.8

NOTES:

1. Fixing error in yards.
2. * indicates largest value recorded for that point.
3. Angle indicates included angle in degrees.

One additional set of ten runs was made. Maximum bearing error was 0.50 degree, or ± 0.25 degree maximum error applied to any given bearing. This magnitude of bearing error for the overall system, even presuming inertial quality heading and primarily digital system components, is believed to be beyond that of any installed aircraft system today and probably beyond today's state-of-the art. Inclusion of data for these runs serves to complete the span of possible system characteristics from the bearing accuracies of current analog systems thru improved current and realizable systems to those beyond that which can be produced at this time. The data pertaining to these ± 0.25 degree runs is displayed in Table IV. The fixing accuracies obtained were on the order of 100 yards at the ten mile range (maximum range) down to 8 to 18 yards at the minimum ranges of 2000 to 3500 yards. Such a capability, if realizable, would be of significant value in airborne local area navigation. The chance of airborne system errors masking or overshadowing actual navigational information would be very small in such a system.

3.2 Results Using Averaging

If one has a number of data points randomly distributed about some mean value, and each of the points is independent, then one would expect the arithmetic average of a number of these points to fall somewhere near the aforementioned mean. Further, the more points used to obtain

TABLE IV

FIXING ERRORS FOR $\pm$0.25 DEGREE BEARING ERRORS

Point	Angle	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8	Run 9	Run 10	Average
43	30	86	18	136	70	*300	44	240	56	36	170	115.6
53	35	*206	116	149	117	202	173	89	37	87	70	124.6
63	40	87	77	19	103	135	53	55	53	53	*181	81.6
74	15	272	66	199	265	342	73	*583	520	145	393	285.8
87	35	42	0	28	*101	31	47	83	88	72	83	57.5
97	45	82	59	79	*131	25	43	114	88	129	114	86.4
107	35	35	55	18	138	81	42	94	153	*168	154	93.8
110	20	52	69	*133	107	52	57	110	77	33	38	72.8
120	30	96	96	35	10	*143	47	90	88	46	28	67.9
131	20	81	100	102	34	44	*178	50	77	77	24	76.7
143	25	135	*148	91	42	85	97	72	76	54	20	82.0
153	25	27	83	20	18	*118	60	18	26	13	18	40.1
175	45	11	11	9	19	15	*21	15	*21	7	15	14.4
207	60	4	5	*16	8	7	7	4	6	14	4	7.5
210	40	13	9	8	18	21	18	21	4	*22	21	15.5
220	45	*29	10	26	19	10	20	23	6	19	17	17.9

NOTES:

1. Fixing error in yards.
2. * indicates largest value recorded for that point.
3. Angle indicates included angle in degrees.

such an average, the more closely it should correspond to the expected mean. If one then has an antenna system capable of taking a number of bearings in rapid succession, and if each bearing is independent of each other bearing, an operationally useful procedure suggests itself. "System accuracy"; i.e., the accuracy with which a given bearing may be obtained, will be improved if one takes a series of bearings in rapid succession, averages them, and then uses this computed "mean bearing" at the mid-time of the observations rather than using any one of the single bearings actually observed. This technique, if implemented, could be used to upgrade a marginally useful system to one that is useful over all ranges of interest. It could also be used to further refine a system already acceptably accurate to one subject to even smaller system fixing errors.

In order to verify the above line of reasoning, random numbers from 0 to 10 were chosen from the table of random numbers in Reference 6. These numbers were grouped by tens, and the simple arithmetic average computed for each group. Over fifty of these "averaged random numbers" were prepared; enough to conduct three additional experimental runs. Figure 5 shows the distribution of these "averaged" numbers on the basis of 0 to 1.00. As can be observed, the distribution is heavily concentrated within $\pm$12 to 15 per cent of the mean; the actual mean of the 510 inputs used is 0.493 which agrees quite closely with the expected value of 0.500. If the distribution of Figure 5

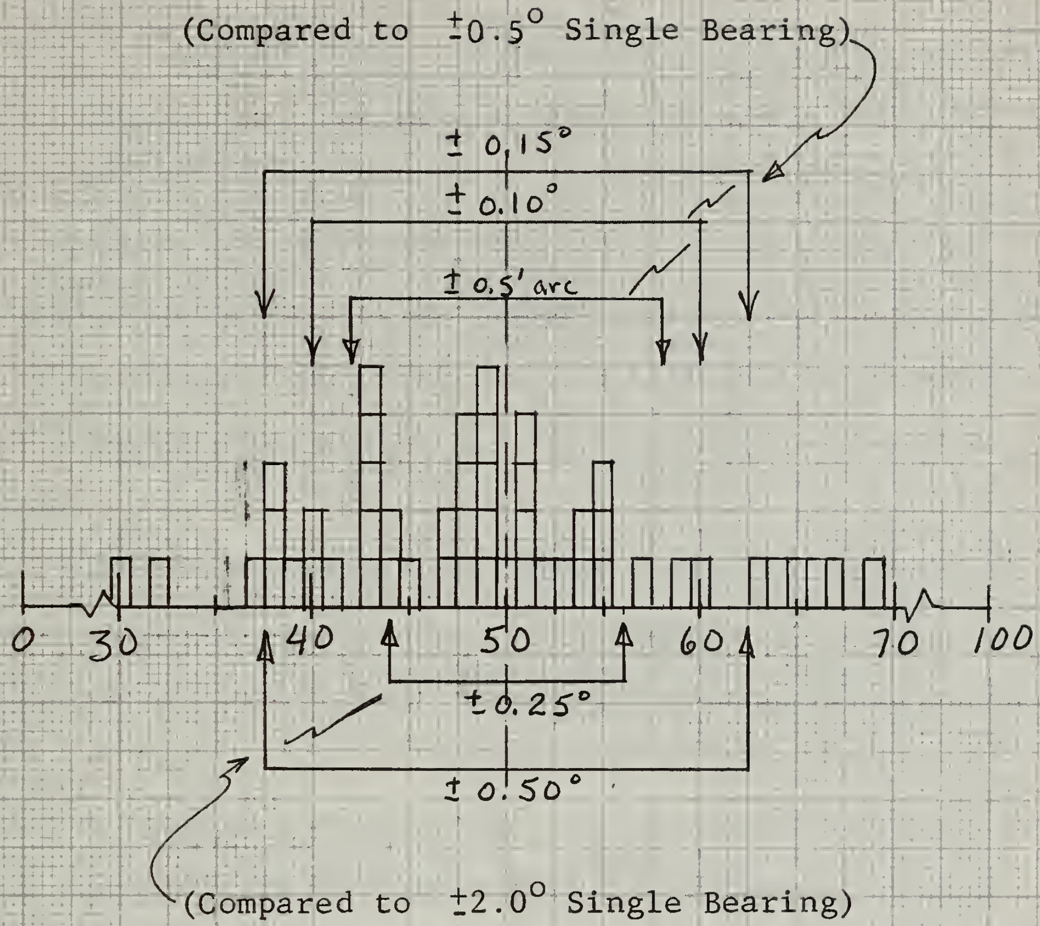


FIGURE 5

DISTRIBUTION OF "AVERAGED" ERROR VALUES

is considered to be bearing error applied to a system having a limiting single bearing error of ± 2.0 degrees it would appear that the results should compare favorably with the runs conducted previously at system bearing errors of ± 0.5 degree, since 82 per cent of the data points fall within this range. It is further noted that 47 per cent (almost half) of the data points fall within the limits that would correspond to ± 0.25 degree single bearing error.

Three experimental runs were conducted to verify these calculations for the ± 2.0 degrees case. Table V shows the point-by-point results of these individual runs plus the mean value of fixing error at each individual point as compared to previous values obtained for the ± 2.0 degrees, ± 0.50 degree and ± 0.25 degree systems. As predicted, the fixing errors of the smoothed ± 2.0 degrees runs compare very closely with fixing errors obtained previously with a ± 0.50 degree system. A system previously judged ". . . useful in at close ranges but most certainly does not produce consistently useful information at medium and long ranges . . . would probably not improve overall system performance sufficiently to justify its inclusion" has by this averaging technique become equivalent to one which ". . . gives a fixing capability . . . which if incorporated into any current aircraft system would enhance significantly the local area navigation capability of that system."

TABLE V

FIXING ERROR FOR ± 2.0 DEGREES (Averaged)

Point	Angle	Averaged Runs				Previous Runs		
		Run 1	Run 2	Run 3	Mean	2.0°	0.50°	0.25°
43	30	160	23	70	84	1170	269	116
53	35	248	55	241	181	854	205	125
63	40	468	33	104	202	712	176	82
74	15	527	892	223	547	1781	721	286
87	35	216	508	12	245	849	277	57
97	45	156	318	261	245	653	231	86
107	35	170	524	193	296	881	222	94
110	20	523	638	570	577	973	226	73
121	30	336	190	119	215	617	159	68
131	20	92	227	279	199	676	155	77
143	25	92	228	100	140	705	143	82
153	25	113	173	34	107	325	99	40
175	45	29	28	31	29	105	27	14
207	60	20	25	8	18	73	14	7
210	40	17	13	28	19	77	19	15
220	45	36	26	9	24	140	31	18

NOTES:

1. Fixing error in yards.
2. Angle indicates included angle in degrees.

The reasoning used above now applied to a system with a basic ± 0.5 degree maximum bearing error indicates that it should be improved by the averaging technique to give fixing accuracies better than that previously obtained with a ± 0.25 degree system. The upper brackets applied to Figure 5 illustrate the anticipated effect of averaging on such a ± 0.50 degree system. In order to verify this conclusion experimentally three runs were made with a ± 0.50 degree system. The same sets of random "averaged" numbers used on the previous ± 2.0 degrees runs were used as the source of random bearing error for these runs. The resulting fixing errors are shown in Table VI, compared with the basic results obtained for values of ± 0.50 degree and ± 0.25 degree maximum bearing error. As predicted the system has again been upgraded, this time from a usable system to one improved sufficiently to be operationally usable at all "local area" ranges under consideration. The success of these two sets of experimental runs illustrates the very real potential of bearing averaging as a means of achieving a capability that can complement and contribute to existing system performance, where only a marginally useful or an unacceptable capability existed before.

3.3 Discussion

Great care must be used in designing an experiment to simulate the real world. One must be certain that the model is sufficiently realistic that conclusions drawn

TABLE VI

FIXING ERROR FOR ± 0.50 DEGREE (Averaged)

Point	Angle	Averaged Runs				Previous Runs	
		Run 1	Run 2	Run 3	Mean	0.50^{o}	0.25^{o}
43	30	46	11	18	25	269	116
53	35	67	13	66	49	205	125
63	40	121	6	26	51	176	82
74	15	134	228	45	136	721	286
87	35	57	133	6	65	277	57
97	45	43	87	60	63	231	86
107	35	43	138	41	74	222	94
110	20	137	160	132	143	226	73
120	30	88	44	28	53	159	68
131	20	20	54	68	47	155	77
143	25	24	54	21	33	143	82
153	25	36	49	13	33	99	40
175	45	5	13	2	7	27	14
207	60	5	6	2	4	14	7
210	40	4	4	7	5	19	15
220	45	9	6	.2	6	31	18

NOTES:

1. Fixing error in yards.
2. Angle indicates included angle in degrees.

relative to behavior of the model, and extrapolations from data obtained experimentally with the model will remain valid when applied to the real world system that was modeled. It has been assumed in this experiment that the navigation and computing portions of the real aircraft system are sufficiently accurate that bearing errors of the order discussed may be attributed to the antenna system; or, alternatively, are at least of sufficiently small magnitude that total "system" bearing errors (or uncertainty) of the orders assumed are not unrealistic. Although values of specific aircraft system error cannot be quoted in this work, the four sets of experimental runs made at assumed error value of ± 5.0 degrees, ± 2.0 degrees, ± 0.50 degree and ± 0.25 degree do cover sufficient span to encompass analog systems, a digital system such as the P3C A-NEW described in Reference 2, and accuracies greater than can be achieved in operating systems today. Further, in order to make use of the averaging technique, the ability is assumed to compute at least ten bearings, average them, examine for compliance with stipulated conditions, then cross with another bearing (being held in memory) to obtain a fix, all within a time interval of no more than 20 seconds. This is well within the capability of current digital systems. One hardware consideration which must be held in mind if a rotating or loop antenna is used is to assure antenna system dynamics and sensitivity such that a series of ten independent bearings can be sampled in the

desired time interval. Subject to the above comments the model used in this research appears to be truly representative of a real aircraft system, and the experimental results obtained may be validly applied to a real situation.

The results set forth in previous paragraphs of this section spell out the details of a frustrating problem and of its potential means of solution. It appears quite obvious that the analog aircraft system with its slow data rate and inherently large and shifting errors will never be able to make use of a bearings-only type of fixing system. Visual overflight of a reference object after some type of homing will remain a firm requirement for local area navigation. Fog, rain and reduced visibility will always hamper such a system. Given an airborne system such as that in the P3C of Reference 2, however, the experimental data indicate that a usable bearings-only navigation aid can be realized. Depending upon the specific capability of the aircraft system, the technique of averaging the value of a group of bearings to obtain a "mean value" offers an attractive means of upgrading significantly the fixing capability of the system so as to make a marginal system usable or an acceptable system even better. The attractiveness of the bearings-only navigation capability lies in its relieving of the aircraft from the necessity to repeatedly overfly the reference point or even, for that matter, to ever again sight it visually once it has been dropped. Hampering tactical constraints can be relaxed,

and the airborne system become truly "all weather," unconstrained by rain or reduced visibility and not required to conform to any artificially contrived and constraining flight pattern. This capability both complements and adds to the increased tactical effectiveness afforded by incorporation of digital data processing in the aircraft system.

3.4 Operational Considerations

A number of additional considerations apply when considering adaptation of the bearings-only technique to a real system. In addition to sorting bearings for angular change of at least 15^{o} (or some other arbitrary minimum) and the arbitrarily imposed minimum time interval of 20 seconds, the system must recognize some bounds on fixing error. A minimum value must be assigned, perhaps of the order of 50 to 75 yards, below which the system will be considered to have remained "fixed" since the last computation; slewing or updating the entire plot for such a small apparent change would not normally be justified. At the other extreme, a maximum value must be assigned, perhaps of the order of 300 to 350 yards, beyond which the system will consider the "apparent" correction factor to be so large as to be unreasonable, and it will be rejected. Without this "upper limit" being defined an occasional out-of-bounds piece of data will cause an unreasonably large and unjustified system "slew" or

repositioning, only to have the entire plot moved right back again at the next computation.

Allowances must also be made for essentially fixed installation biases and associated antenna pattern distortions due to placement of antenna(e) and the configuration of the airplane. In the case of rotating or loop antenna systems there will probably be an additional bias the magnitude of which is dependent upon the angular bearing rate, and which will be a "lead" or a "lag" depending upon the clockwise or counterclockwise sense of the relative motion. Both of these system biases can be predicted for a given installation, and a "table" correction (or corrections) applied during the digital computation. One additional feature, a bank angle cutout, will probably be required. Because of the gross antenna pattern distortion and blanking of entire quadrants during steeply banked turns, some provision must be made to interrupt or "cut out" the bearing sampling function whenever the aircraft is banked beyond some predetermined amount. This cut out may be an actual "hardware" interrupt, or the system can be programmed to ignore bearing information under certain specified conditions. The function and all the other operational considerations discussed in this section can be provided for in designing and implementing a specific system and its associated computer software program.

CHAPTER 4

CONCLUSIONS AND RECOMMENDATIONS

4.1 Conclusions

Based upon the calculations involved and the experimental data obtained with the simulation model, several conclusions can be drawn concerning the use of bearings-only fixing as an element of an aircraft local area navigation system. These conclusions are:

1. It is not within the state-of-the-art of today's airborne systems to incorporate an operationally useful bearings-only local navigation capability based upon cross-plotting of individual bearings. The experimental data show the runs at ± 5.0 degrees bearing error, representative of an analog system, to be unacceptable for use in bearings-only fixing. Fixing errors are on the order of hundreds of yards to several thousand yards. The next set of experimental runs, those corresponding to maximum bearing errors of ± 2.0 degrees, represent a significant improvement in fixing accuracy. As shown in Table II, the expected fixing errors for short range, close-in data points are small, 75 to 140 yards, and probably usable; however, at

medium and longer ranges out to ten nautical miles (20,000 yards) the fixing error quickly becomes hundreds of yards and is unacceptable. It is not likely that such a limited capability would be incorporated into an otherwise useful system. It is believed that the ± 2.0 degrees bearing accuracy approaches the approximate minimum total system error that can be achieved with any existing aircraft system today.

2. <u>In order to be operationally useful as part of a local area navigation system a single bearing system must have a bearing accuracy of approximately ± 0.50 degree.</u> Examination of the data of Tables I, II and III shows an orderly, increasing accuracy of fixing as system errors decrease from ± 5.0 degrees through ± 2.0 degrees to ± 0.50 degree. This so-called "one degree system" is the first one to show sufficient fixing accuracy to be of value at the long ranges up to 20,000 yards, and it becomes extremely accurate with fixing errors on the order of from 15 to 30 yards in at minimum ranges. If a system could be produced to this order of accuracy it would be usable as a bearings-only fixing system at all ranges in question in the local area.

3. <u>If single bearings are averaged in groups of ten or more, a system which is basically accurate to ± 2.0 degrees or better can be upgraded to a usable system. Such a system appears to be within today's state-of-the-art.</u> Looking at the data of Table V and Table VI it can

be seen that in both averaging cases the fixing accuracies improve significantly, and for the cases shown, become as good or better than that obtained at ± 0.50 degree single bearing fixing error. Assuming a basically digital aircraft system with a doppler-inertial navigation capability and averaging of groups of ten bearings, a ± 2.0 degrees system should prove to be operationally useful throughout the local area.

4.2 Recommendations

The purpose of this research was to investigate the bearings-only fixing technique by means of a computer simulation model, and to attempt to determine the order of system bearing accuracy that would be required in order to generate useful fixing information. Additionally, bearing averaging was superimposed on the basic model to determine the degree of improvement that would be introduced by that technique.

It has been concluded, based on the results of the simulation runs, that system bearing accuracy of approximately ± 0.50 degree is required for usable bearings-only fixing, that today's best systems will give accuracies of approximately ± 2.0 degrees, and that bearing averaging will upgrade such a system to the equivalent of a ± 0.50 degree system. It is recommended that a simple aircraft system be fabricated, using a loop antenna and reasonable care in selection of components. This could be based on

the present direction finding system in the P3 aircraft plus any modifications known to be feasible and desirable. A series of simple flights, straight line or arcs of constant radius, should be flown to gather data as to the potential bearing accuracy of such a system.

A simple computer program should be written to carry out the bearings-only fixing technique as described in this thesis. Other commitments permitting it could be flight tested, in conjunction with the hardware described above, in the developmental A-NEW system. Bearing averaging should be included to verify the predicted improvement. Further research is recommended into the matter of minimum acceptable angular change between two adjacent bearing lines. Although 15 degrees was set arbitrarily in this experimental model, only one intersection, Point 74, worked out to be a 15 degree angle, and it was consistently much worse than other surrounding points. Possibly a range-angle criteria would be better than one based on angle alone. Additional simulation is recommended along this line of investigation.

The P3C A-NEW system frequently referred to represents the most capable system available today. It is recommended that serious consideration be given to incorporation of the bearings-only capability in that system, and that further development be pursued toward eventual use in future aircraft systems.

APPENDIX A

AN EXAMPLE OF A TYPICAL CTSS COGO-90 COMPUTER PROGRAM

This appendix contains a sample COGO-90 problem conducted using the M.I.T. Compatible Time-Sharing System (CTSS). X-Y coordinates assigned to points in this sample problem correspond to those of the same points in the experiment, as shown in Figure 3. Typical experimental values are used for the bearings which are cross-plotted to give two bearings-only fixes, designated Point 43 and Point 120 according to the convention of this thesis.

The following COGO commands are utilized in this sample problem:

Clear: Clear memory table of any previously stored coordinates.

Store: Store X-Y coordinates of designated point.

Az/intersect: Locate a new point by forming the intersection of the two bearing lines having azimuths as indicated.

Distance: Compute the distance between the points indicated.

Dump: List the X-Y coordinates of all designated points within the stated limits.

Notes Concerning Sample Problem

1. Lower case indicates input to CTSS, upper case is output from CTSS.
2. A command need not be repeated in a series of inputs, each utilizing the same command.
3. Explanatory comments may appear to the right of any input line.

The reader is referred to Reference 5 for more complete details concerning COGO-90 as used with CTSS.

```
start           sample cogo-90 problem
1                                 SAMPLE COGO-90 PROBLEM


READY
clear           0 999       clear storage
READY
store           0 0 0        store point 0
READY
                3 -20000 0   point 3
READY
                4 -17321 10000       pt 4
READY
                10 -2084 11818   pt 10
READY
                12 -6428 7660    pt 12
READY


az/intersect  43 4 330 06 00 3 359 54 00
 PT= 43   YCOORD=    130.631   XCOORD=   -35.155
READY
                120 12 309 48 00 10 280 12 00
 PT=120   YCOORD=     66.711   XCOORD=  -135.208
READY


distance        0 43   didtance from pt 0
 FROM POINT    0 TO POINT  43DISTANCE=   135.278
READY
                0 120    dist. from pt 0
 FROM POINT    0 TO POINT 120DISTANCE=   150.770
READY


dump            0 120       list all points
 PT=  0   YCOORD=      0.        XCOORD=     0.
 PT=  3   YCOORD=-20000.000    XCOORD=     0.
 PT=  4   YCOORD=-17321.000    XCOORD= 10000.000
 PT= 10   YCOORD= -2084.000    XCOORD= 11818.000
 PT= 12   YCOORD= -6428.000    XCOORD=  7660.000
 PT= 43   YCOORD=   130.631    XCOORD=   -35.155
 PT=120   YCOORD=    66.711    XCOORD=  -135.208
READY


finish                     end of sample problem
```

APPENDIX B

INPUT DATA FOR CONDUCT OF ± 2.0 DEGREES BEARING ERROR COMPUTER RUNS

The following pages contain the data inputted to the computer in the conduct of ten experimental runs with maximum bearing errors of ± 2.0 degrees, and three additional runs with ± 2.0 degrees "averaged" errors. Prior to the start of each run, or series of runs, the X-Y coordinates of the basic reference points along the flight path, Point 3 through Point 22, plus the location of the reference buoy, Point 0, were supplied to the computer. This data is reproduced on the following page for reference.

BASIC REFERENCE POINTS

POINT	X COORDINATE	Y COORDINATE	DISTANCE
0	0.	0.	0
3	0.	-20,000.	20,000
4	10,000.	-17,321.	20,000
5	11,472.	-16,383.	20,000
6	12,856.	-15,321.	20,000
7	14,142.	-14,142.	20,000
8	17,727.	-3,126.	18,000
9	16,000.	0.	16,000
10	11,818.	-2,084.	12,000
11	8,660.	-5,000.	10,000
12	7,660.	-6,428.	10,000
13	6,428.	-7,660.	10,000
14	2,329.	-8,693.	9,000
15	776.5	-2,898.	3,000
17	1,732.	-1,000.	2,000
20	1,732.	1,000.	2,000
21	1,026.	2,819.	3,000
22	1,294.	4,830.	5,000

KEY TO TABLES

1. For runs 1 through 10, bearings are in degrees and tenths (nearest 6' of arc); for averaged runs bearings are in degrees and minutes.

2. For runs 1 through 10, f is a random factor from 0.0 to 4.0 equivalent to -2.0^{o} to $+2.0^{o}$; for averaged runs 1, 2 and 3, f as a random factor from 00 to 100 equivalent to -2.0^{o} to $+2.0^{o}$.

3. Points numbered 16, 18 and 19 are omitted because they were not used due to limiting criteria.

APPENDIX B

INPUT DATA FOR CONDUCT OF ± 2.0 DEGREES BEARING ERROR COMPUTER RUNS

Runs 1 Through 5

Point	Bearing	Run 1		Run 2		Run 3		Run 4		Run 5	
		f	Bearing	f	Bearing	f	Bearing	f	Bearing	f	Bearing
3	360	2.8	000.8	1.4	359.4	3.0	001.0	0.3	358.3	1.2	359.2
4	330	1.3	329.3	0.1	328.1	3.9	331.9	3.3	331.3	2.2	330.2
5	325	1.3	324.3	2.3	325.3	0.1	323.1	1.2	324.2	0.5	323.5
6	320	3.4	321.4	0.7	318.7	3.7	321.7	2.6	320.6	2.8	320.8
7	315	2.8	315.8	3.5	316.5	2.9	315.9	3.3	316.3	0.1	313.1
8	280	2.9	280.9	1.0	279.0	2.7	280.7	3.2	281.2	2.2	280.2
9	270	3.8	271.8	2.7	270.7	3.6	271.6	3.3	271.3	0.3	268.3
10	280	3.9	281.9	1.1	279.1	1.6	279.6	3.5	281.5	2.6	280.6
11	300	0.9	298.9	2.7	300.7	2.8	300.8	3.7	301.7	1.0	299.0
12	310	3.4	311.4	3.6	311.6	0.7	308.7	0.0	308.0	2.5	310.5
13	320	1.2	319.2	1.0	319.0	3.4	321.4	0.1	318.1	2.0	320.0
14	345	3.8	346.8	3.9	346.9	0.8	343.8	2.8	345.8	0.2	343.2
15	345	2.8	345.8	3.3	346.3	3.6	346.6	2.1	345.1	0.3	343.3
17	300	0.5	298.5	3.8	301.8	1.5	299.5	3.4	301.4	3.4	301.4
20	240	3.0	241.0	0.3	238.3	0.8	238.8	3.2	241.2	0.9	238.9
21	200	2.9	200.9	1.5	199.5	2.0	200.0	1.6	199.6	1.0	199.0
22	195	2.7	195.7	0.3	193.3	3.2	196.2	0.4	193.4	0.7	193.7

APPENDIX B

INPUT DATA FOR CONDUCT OF ± 2.0 DEGREES BEARING ERROR COMPUTER RUNS

Runs 6 Through 10

Point	Bearing	Run 6		Run 7		Run 8		Run 9		Run 10	
		f	Bearing	f	Bearing	f	Bearing	f	Bearing	f	Bearing
3	360	3.6	001.6	2.4	000.4	2.6	000.6	2.2	000.2	1.9	359.9
4	330	0.2	328.2	0.3	328.3	1.4	329.4	1.2	329.2	3.5	331.5
5	325	3.3	326.3	1.1	324.1	0.5	323.5	3.1	326.1	3.9	326.9
6	320	1.5	319.5	0.0	318.0	2.1	320.1	2.5	320.5	3.1	321.1
7	315	0.4	313.4	0.8	313.8	0.4	313.4	1.1	314.1	1.6	314.6
8	280	1.4	279.4	0.1	278.1	3.3	281.3	0.1	278.1	3.4	281.4
9	270	1.1	269.1	2.9	270.9	3.5	271.5	0.7	268.7	3.1	271.1
10	280	3.0	281.0	2.0	280.0	3.6	281.6	0.0	278.0	2.8	280.8
11	300	1.3	299.3	2.6	300.6	1.4	299.4	2.2	300.2	1.0	299.0
12	310	1.8	309.8	2.6	310.6	1.0	309.0	2.1	310.1	1.9	309.9
13	320	1.7	319.7	0.0	318.0	2.1	320.1	2.1	320.1	1.9	319.9
14	345	0.2	343.2	0.2	343.2	0.7	343.7	3.1	346.1	3.4	346.4
15	345	2.1	345.1	0.4	343.4	3.2	346.2	1.8	344.8	1.8	344.8
17	300	3.4	301.4	1.4	299.4	0.0	298.0	3.6	301.6	1.1	299.1
20	240	0.3	238.3	2.6	240.6	2.6	240.6	3.3	241.3	2.3	240.3
21	200	2.6	200.6	3.2	201.2	3.9	201.9	3.1	201.1	1.7	199.7
22	195	3.4	196.4	2.3	195.3	3.8	196.8	1.7	194.7	1.8	194.8

APPENDIX B

INPUT DATA FOR CONDUCT OF ± 2.0 DEGREES BEARING ERROR COMPUTER RUNS

Averaged Runs 1 Through 3

Point	Bearing	Average Run 1		Average Run 2		Average Run 3	
		f	Bearing	f	Bearing	f	Bearing
3	360	55	000-12	49	359-58	54	000-10
4	330	49	329-58	50	330-00	55	330-12
5	325	44	324-46	47	324-53	63	325-31
6	320	32	319-17	48	319-55	49	319-58
7	315	39	314-34	66	315-38	51	315-02
8	280	47	279-53	43	279-43	51	280-02
9	270	43	269-43	49	269-58	67	270-41
10	280	30	279-12	38	279-31	64	280-34
11	300	53	300-07	69	300-46	40	299-36
12	310	51	310-02	49	309-58	65	310-36
13	320	57	320-17	59	320-22	54	320-10
14	345	52	345-05	45	344-48	48	344-55
15	345	43	344-43	38	344-31	51	345-02
17	300	37	299-29	60	300-24	48	299-55
20	240	38	239-31	42	239-41	44	239-46
21	200	43	199-43	48	199-55	55	200-12
22	195	40	194-36	43	194-43	49	194-58

APPENDIX C

OUTPUT DATA FROM ±2.0 DEGREES BEARING ERROR COMPUTER RUNS

Appendix C contains selected data from the computer output for the ten experimental runs with maximum bearing error of ±2.0 degrees, and the three additional runs with ±2.0 degrees "averaged" bearing error. The original data have been retained, and are available from the author upon request.

The computer output format lists all coordinates and distances to three decimal places. These values were rounded off to the nearest whole number for all computations.

RUN 1

Point	X Coordinate	Y Coordinate	Distance
43	266.319	-927.648	965.120
53	268.222	-791.324	835.546
63	285.222	426.112	512.760
74	-1341.169	1779.664	2228.438
87	135.024	261.665	294.449
97	-102.614	506.033	516.332
107	-7.241	407.959	408.023
110	-1833.791	792.872	1997.857
120	-121.103	431.953	448.608
131	10.436	-225.195	225.436
143	466.926	-754.043	886.905
153	137.390	-372.272	396.815
175	67.553	-96.281	117.616
207	-90.714	-10.349	91.303
210	-44.676	15.171	47.182
220	-62.099	5.513	62.343

RUN 2

Point	X Coordinate	Y Coordinate	Distance
43	-199.680	-934.564	955.657
53	-214.648	494.630	539.196
63	-204.709	-454.335	498.323
74	-3646.791	4603.441	5872.882
87	1204.905	-509.170	1308.071
97	542.511	188.846	574.440
107	1064.906	-361.642	1124.637
110	768.007	-314.087	829.751
120	774.979	-315.204	836.628
131	-731.869	576.472	931.640
143	502.305	-843.280	981.545
153	187.264	-480.868	516.044
175	61.294	35.879	71.023
207	116.019	1.949	116.036
210	3.913	-67.290	67.404
220	158.971	28.476	161.501

RUN 3

Point	X Coordinate	Y Coordinate	Distance
43	361.838	729.633	814.426
53	322.342	-1533.078	1566.599
63	357.905	504.303	618.399
74	1136.753	-721.651	1346.472
87	270.295	172.464	320.629
97	4.460	446.783	446.805
107	621.861	-190.324	650.334
110	579.721	-183.197	607.978
120	-326.021	-30.003	327.398
131	350.031	-47.274	353.077
143	-488.166	1003.715	1116.132
153	-10.462	405.307	405.442
175	105.038	-79.511	131.739
207	24.633	-34.020	42.002
210	-22.887	-62.799	66.840
220	-149.820	-139.673	204.828

RUN 4

Point	X Coordinate	Y Coordinate	Distance
43	-657.266	2144.853	2243.299
53	-604.327	361.207	704.046
63	-626.025	1092.255	1258.939
74	368.678	270.932	457.523
87	321.423	320.381	453.825
97	286.841	356.569	457.624
107	398.987	239.215	465.204
110	67.908	306.573	314.004
120	-1321.754	589.302	1447.173
131	-1699.328	1398.039	2200.507
143	354.888	-891.419	959.465
153	195.246	-713.495	739.727
175	-11.730	64.375	65.435
207	8.095	52.274	52.897
210	48.779	74.640	89.166
220	178.046	145.705	230.066

RUN 5

Point	X Coordinate	Y Coordinate	Distance
43	-288.275	643.302	704.940
53	-272.146	-511.729	579.594
63	-290.437	798.095	849.299
74	1293.476	-2118.586	2482.234
87	-1285.630	294.896	1319.018
97	-448.793	-488.202	663.141
107	-1383.636	386.607	1436.633
110	-891.594	294.525	938.981
120	-20.125	131.434	132.966
131	314.179	-373.845	488.333
143	-461.983	551.147	719.160
153	-144.941	173.312	225.931
175	-134.795	139.494	193.980
207	84.066	5.901	84.273
210	50.376	-14.422	52.400
220	123.867	29.911	127.427

RUN 6

Point	X Coordinate	Y Coordinate	Distance
43	502.690	-2003.441	2065.544
53	558.138	-18.388	558.441
63	533.695	-893.461	1040.722
74	-635.641	-167.506	657.341
87	-739.992	-68.825	743.186
97	-538.027	-259.813	597.475
107	-1306.819	467.196	1387.821
110	-963.543	400.470	1043.452
120	-405.638	292.024	499.821
131	96.876	-194.609	217.388
143	-421.301	416.404	592.357
153	39.032	-126.401	132.290
175	-11.730	64.375	65.435
207	103.355	-5.873	103.522
210	-77.868	-117.799	141.209
220	-180.951	-181.464	256.267

RUN 7

Point	X Coordinate	Y Coordinate	Distance
43	130.271	-1340.579	1346.893
53	134.596	-721.021	733.476
63	131.330	-1188.847	1196.078
74	-832.085	217.605	860.069
87	27.865	-607.055	607.694
97	-881.700	265.185	920.716
107	-741.284	130.530	752.689
110	293.149	-51.868	297.702
120	202.093	-35.812	205.241
131	-1237.439	853.317	1503.132
143	-208.043	-289.941	356.858
153	101.481	-633.701	641.775
175	-109.888	37.848	116.224
207	-42.714	-0.002	42.714
210	-74.326	-17.814	76.431
220	-24.534	10.243	26.586

RUN 8

Point	X Coordinate	Y Coordinate	Distance
43	201.564	-752.786	779.304
53	197.443	-1146.340	1163.219
63	207.422	-193.499	283.664
74	478.550	-1221.141	1311.562
87	-1588.545	733.611	1749.760
97	-1291.578	452.783	1368.644
107	-1499.870	649.755	1634.562
110	-1290.418	606.760	1425.951
120	-937.849	534.389	1079.413
131	234.206	-252.319	344.263
143	-340.085	434.523	551.786
153	81.919	-70.187	107.875
175	96.738	-130.518	162.460
207	-94.183	-29.003	98.548
210	-126.123	-47.000	134.596
220	-189.177	-82.529	206.395

RUN 9

Point	X Coordinate	Y Coordinate	Distance
43	67.514	-659.135	662.584
53	71.846	582.184	586.601
63	70.474	189.056	201.764
74	-153.020	-289.188	327.177
87	200.323	-631.599	662.606
97	-74.987	-364.806	372.433
107	-17.351	-420.659	421.017
110	1049.505	-570.596	1194.588
120	634.900	-512.327	815.828
131	-20.266	52.025	55.832
143	242.626	-262.390	357.374
153	-27.289	60.425	66.301
175	-34.432	86.714	93.300
207	11.848	58.243	59.435
210	-52.981	22.750	57.659
220	47.239	77.619	90.864

RUN 10

Point	X Coordinate	Y Coordinate	Distance
43	-36.961	1164.741	1165.327
53	-37.148	1271.962	1272.504
63	-36.073	656.292	657.282
74	1510.911	-1686.077	2264.002
87	-821.536	614.024	1025.645
97	-520.535	317.197	609.566
107	-460.807	258.297	528.262
110	-1017.985	364.584	1081.302
120	-300.064	227.634	376.637
131	273.478	-351.284	445.186
143	326.010	-413.668	526.691
153	-5.414	-20.090	20.807
175	-1.291	-35.265	35.288
207	-42.650	-12.244	44.373
210	26.364	27.121	37.824
220	24.790	26.223	36.086

RUN 1 (Averaged)

Point	X Coordinate	Y Coordinate	Distance
43	69.313	-143.671	159.517
53	68.983	-238.172	247.961
63	68.199	-462.582	467.583
74	253.188	-461.724	526.586
87	-215.682	0.106	215.682
97	-134.556	-79.801	156.440
107	-51.548	-161.563	169.587
110	462.313	-244.790	523.121
120	261.016	-212.187	336.382
131	87.517	-27.383	91.701
143	-11.064	91.286	91.954
153	-26.601	109.991	113.162
175	10.792	-26.850	28.938
207	-1.039	-20.161	20.188
210	11.055	-13.042	17.096
220	36.366	1.858	36.414

RUN 2 (Averaged)

Point	X Coordinate	Y Coordinate	Distance
43	-11.667	19.680	22.879
53	-11.625	-53.587	54.833
63	-11.638	-31.192	33.293
74	-446.183	772.281	891.906
87	477.842	-172.398	507.990
97	318.149	-9.136	318.281
107	490.159	-184.991	523.906
110	604.131	-204.097	637.675
120	141.276	-126.504	189.637
131	-89.301	208.723	227.024
143	-89.678	209.179	227.591
153	-52.769	164.616	172.867
175	-13.903	24.313	28.008
207	24.699	1.666	24.755
210	-0.024	-12.791	12.791
220	26.014	2.435	26.128

RUN 3 (Averaged)

Point	X Coordinate	Y Coordinate	Distance
43	58.289	38.167	69.674
53	58.858	233.546	240.848
63	57.926	-86.887	104.425
74	165.796	-149.550	223.279
87	7.365	9.066	11.680
97	-176.276	192.921	261.327
107	-127.807	144.394	192.832
110	-526.814	218.826	570.454
120	25.127	115.866	118.559
131	200.456	-194.320	279.183
143	-38.769	92.466	100.265
153	11.682	31.985	34.052
175	24.981	-17.762	30.652
207	5.191	-6.374	8.220
210	-18.644	-20.265	27.537
220	0.364	-9.187	9.194

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